

## MLE for n-gram LM

$$\max_{\theta} \ell(\theta) = \sum_{i=1}^N \sum_{j=1}^n \log P(x_j | x_{j-1})$$

$= w \in V$   
 $= w' \in V$

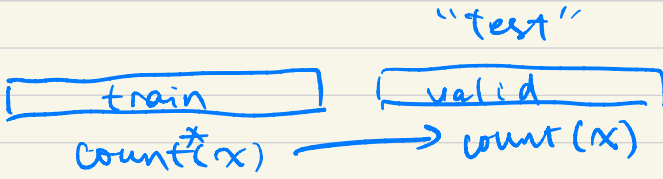
$\theta_{w|w'}$

$$\text{s.t. } \sum_{w \in V} \theta_{w|w'} = 1 \quad \forall w' \in V$$
$$\theta_{w|w'} \geq 0$$

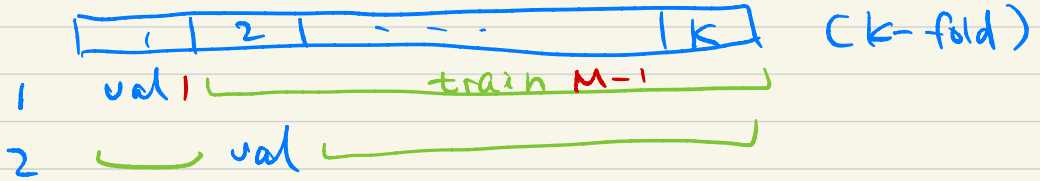
$$L(\theta, \lambda) = \ell(\theta) - \lambda \left( \sum_{w \in V} \theta_{w|w'} - 1 \right)$$

$$\frac{\partial L}{\partial \theta_{w|w'}} = 0$$

Idea:

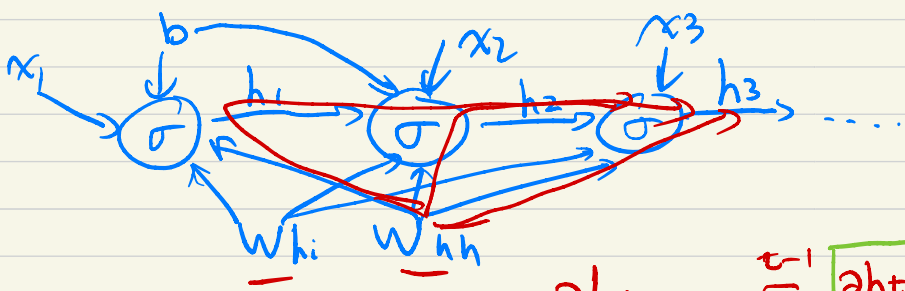


Leave one out (LOO) validation



$$h_t = \sigma(W_{hh}h_{t-1} + W_{hi}x_t + b)$$

$$\frac{\partial \mathcal{L}_t}{\partial W_{hh}} = \frac{\partial \mathcal{L}_t}{\partial o_t} \frac{\partial o_t}{\partial h_t} \boxed{\frac{\partial h_t}{\partial W_{hh}}}$$



$$\frac{\partial h_t}{\partial W_{hh}} = \sum_{i=1}^{t-1} \boxed{\frac{\partial h_t}{\partial h_i}} \frac{\partial h_i}{\partial W_{hh}}$$

$$\begin{aligned} \frac{\partial h_t}{\partial h_i} &= \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial h_{t-2}} \dots \frac{\partial h_{i+1}}{\partial h_i} \\ &= \prod_{j=i}^{t-1} \sigma' W_{hh} \end{aligned}$$

$$W_{hh}^k = Q \Lambda^k Q$$