Distributed representation of text

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Last week

Generative vs discriminative models for text classification

- (Multinomial) naive Bayes

What’s the key assumption?

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Feature vector of text input

- BoW representation
- N-gram features (usually $n \leq 3$)

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🤔 What text representation have we learned?

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Idea: Represent a word by its neighbors.

Step 1: Choose the context

What are the neighbors?

Example:

- word × document

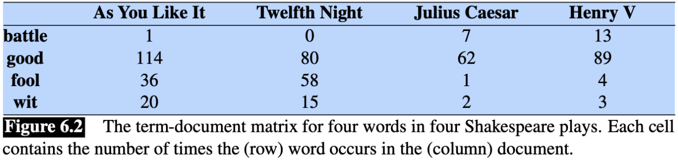

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Figure 6.2 The term-document matrix for four words in four Shakespeare plays. Each cell contains the number of times the (row) word occurs in the (column) document.

Figure: Jurafsky and Martin.

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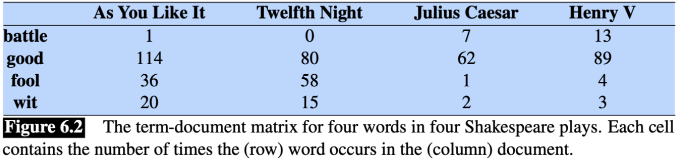

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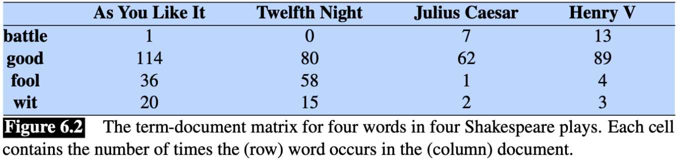

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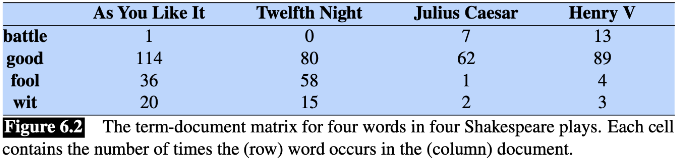

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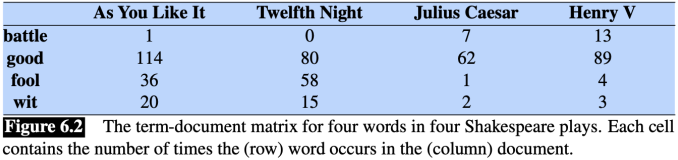

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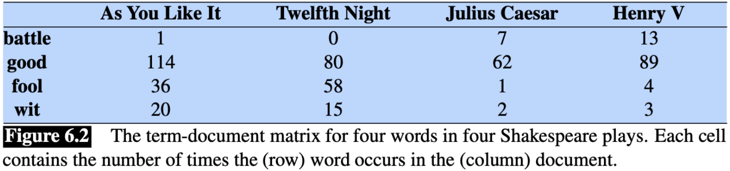
Construct a matrix where

- Row and columns represent two sets of objects
- Each entry is the co-occurence counts of the two objects

Distance functions

Now we already have a representation for each document / word!



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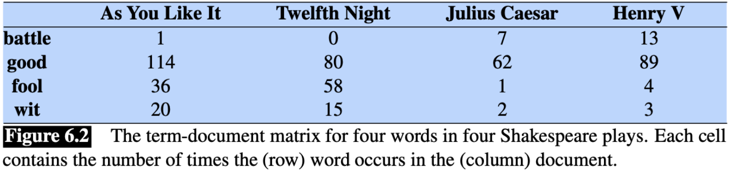

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How do we compute similarities between documents?

Euclidean distance

For $a, b \in \mathbb{R}^d$,

$$d(a, b) = \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2} \ .$$

Assume a and b are BoW vectors. What if b repeats each sentence in a twice?

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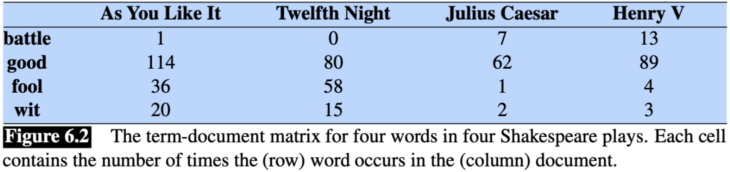

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Cosine similarity

For $a, b \in \mathbb{R}^d$,

$$\text{sim}(a, b) = \frac{a \cdot b}{\|a\| \|b\|}$$

Defines angle between two vectors ($=\cos\alpha$ in 2D)

Example application: information retrieval

Given a set of documents and a query, use the BoW representation and cosine similarity to find the most relevant document.

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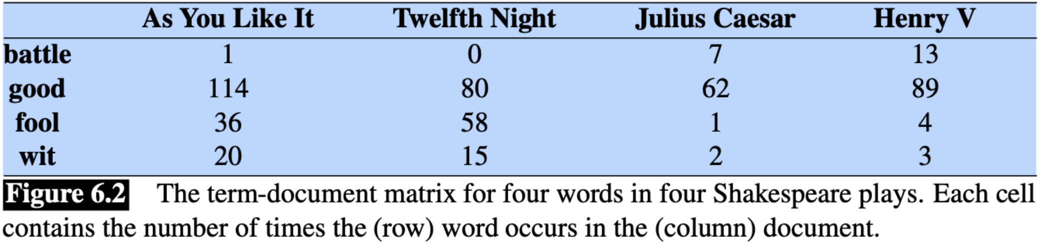
What are potential problems?

Q: What are good books on great battles?



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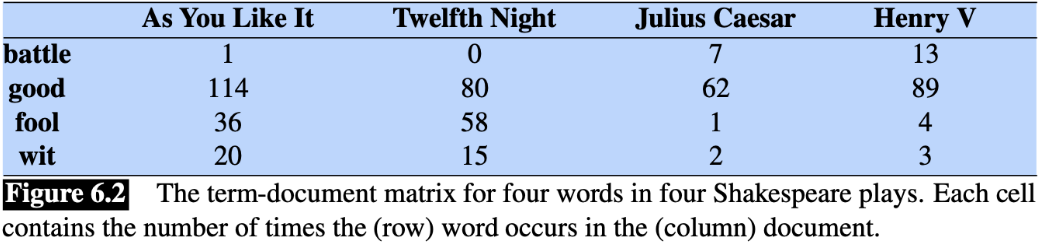

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Similarity may be dominated by common words!

Step 2: Reweight counts

Key idea: upweight words that carry more information about the document

Construct a feature map $\phi\colon$ document $\rightarrow \mathbb{R}^{|\mathcal{V}|}$

TFIDF weight for token w:

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$$\phi_w(d) = \underbrace{\text{count}(w, d)}_{\mathrm{tf}(w,d)} \times$$

- **Term frequency (TF)**: count of the word in the document (same as BoW)

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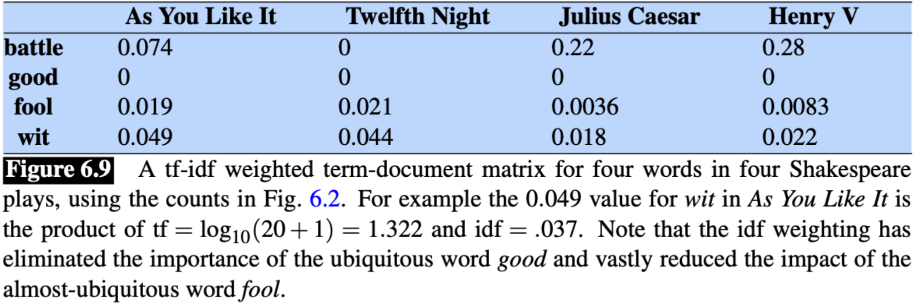
- **Term frequency (TF)**: count of the word in the document (same as BoW)
- Reweight by **inverse document frequency (IDF)**: how specific is the word to any particular document
- Higher weight on frequent words that only occur in a few documents

TFIDF example



	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Figure 6.9 A tf-idf weighted term-document matrix for four words in four Shakespeare plays, using the counts in Fig. 6.2. For example the 0.049 value for *wit* in *As You Like It* is the product of $\text{tf} = \log_{10}(20+1) = 1.322$ and $\text{idf} = .037$. Note that the idf weighting has eliminated the importance of the ubiquitous word *good* and vastly reduced the impact of the almost-ubiquitous word *fool*.

Figure: From Jurafsky and Martin.

Why do some words have zero weights?

An alternative way to reweighting using pointwise mutual information

$$\mathsf{PMI}(x; y) \overset{\text{def}}{=} \log \frac{p(x, y)}{p(x)p(y)} = \log \frac{p(x \mid y)}{p(x)} = \log \frac{p(y \mid x)}{p(y)}$$

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$$\hat{p}(x \mid y) = \frac{\mathrm{count}(x, y)}{\sum_{x' \in \mathcal{X}} \mathrm{count}(x', y)}$$ how often does x occur in the neighborhood of y

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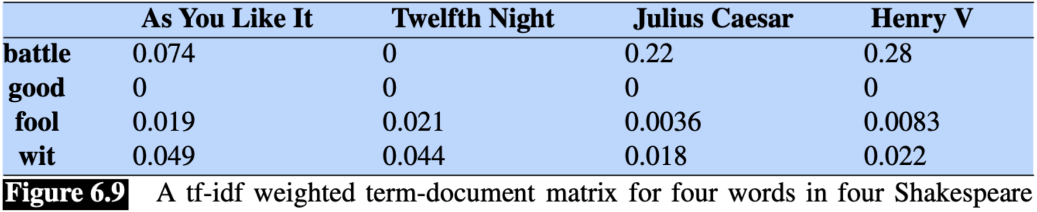
- **Positive PMI**: $\mathrm{PPMI}(x; y) \overset{\mathrm{def}}{=} \max(0, \mathrm{PMI}(x; y))$
- Application in NLP: measure association between words

Step 3: Dimensionality reduction

What’s the size of this matrix?



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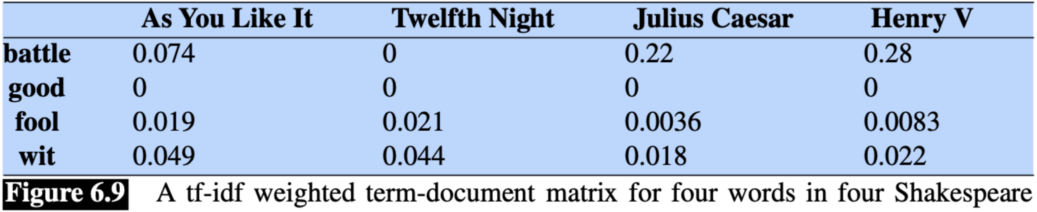

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Motivation: want a lower-dimensional, dense representation for efficiency

Step 3: Dimensionality reduction

Recall **SVD**: a $m \times n$ matrix $A_{m \times n}$ (e.g., a word-document matrix), can be decomposed to

$$U_{m \times m} \Sigma_{m \times n} V^T_{n \times n} \ ,$$

where U and V are orthogonal matrices, and Σ is a diagonal matrix.

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Interpretation:

$$AA^T = (U\Sigma V^T)(V\Sigma U^T) = U\Sigma^2 U^T .$$

- σ_i^2 are eigenvalues of AA^T
- Connection to PCA: If columns of A have zero mean (i.e. AA^T is the covariance matrix), then columns of U are principle components of the column space of A.

SVD for the word-document matrix

$$A = \begin{array}{c c c c c} d_1 & d_2 & \cdots & d_n & \\ \left(\begin{array}{c} 12 \\ 7 \\ 4 \\ 9 \end{array}\right. & \begin{array}{c} 5 \\ 10 \\ 8 \\ 3 \end{array} & \begin{array}{c} \cdots \\ \cdots \\ \cdots \\ \cdots \end{array} & \left.\begin{array}{c} 8 \\ 3 \\ 6 \\ 7 \end{array}\right) & \begin{array}{c} \text{US} \\ \text{gov} \\ \text{gene} \\ \text{lab} \end{array} \end{array}$$

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$$= \begin{pmatrix} 0.50 & 0.02 & \cdots \\ 0.60 & 0.03 & \cdots \\ 0.01 & 0.72 & \cdots \\ 0.02 & 0.84 & \cdots \end{pmatrix}_{4\times 4} \begin{pmatrix} 15 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 10 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 5 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 2 & \cdots & 0 \end{pmatrix}_{4\times n} \begin{pmatrix} 0.50 & 0.60 & \cdots \\ 0.64 & 0.48 & \cdots \\ 0.12 & 0.24 & \cdots \\ 0.36 & 0.12 & \cdots \end{pmatrix}^T_{n\times n}$$

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- u_i are document clusters and v_i are word clusters
- Take top-k components to obtain word vectors: $W = U_k \Sigma_k$ (or just U_k)

SVD for the word-document matrix

Computing the dense word vectors:

- Run **truncated SVD** of the word-document matrix $A_{m \times n}$
- Each row of $U_{m \times k}\Sigma_k$ corresponds to a word vector of dimension k
- Each coordinate of the word vector corresponds to a cluster of documents (i.e., topics such as politics, music, etc.)

Summary

Count-based word embeddings

1. Design the matrix, e.g. word × document, people × movie.
2. Reweight the raw counts, e.g. TFIDF, PPMI.
3. Reduce dimensionality by truncated SVD.
4. Use word/person/etc. vectors in downstream tasks.

Key idea:

- Intuition: Represent an object by its connection to other objects.
- Lexical semantics: the word meaning can be represented by the context it occurs in.
- Linear algebra: Infer clusters (e.g., concepts, topics) using co-occurence statistics

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Distributional hypothesis: Similar words occur in similar contexts

- Predict the context given a word $f:$ word $\rightarrow$ context
- Words that tend to occur in same contexts will have similar representation

The skip-gram model

Task: given a word, predict its neighboring words within a window

The quick brown fox jumps over the lazy dog

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Assume **conditional independence** of the context words:

$$p(w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k} \mid w_i) = \prod_{j=i-k, j \neq i}^{i+k} p(w_j \mid w_i)$$

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How to model $p(w_j \mid w_i)$? Multiclass classification

The skip-gram model

Use the softmax function to predict context words from the center word

$$p(w_j \mid w_i) = \frac{\exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}$$

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- For each word w, learn two vectors.
- Learn parameters by MLE and SGD (Is the objective convex?)
- ϕ_{wrd} is taken as the word embedding

Negative sampling

Challenge in MLE: computing the normalizer is expensive (try calculate the gradient)!

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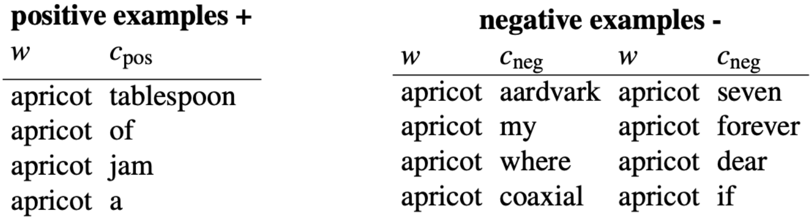
Key idea: solve a binary classification problem instead

Is the (word, context) pair real or fake?



positive examples +

w	c_{pos}
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

w	c_{neg}	w	c_{neg}
apricot	aardvark	apricot	seven
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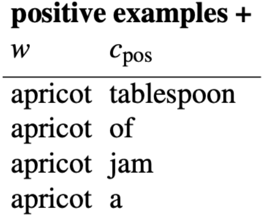

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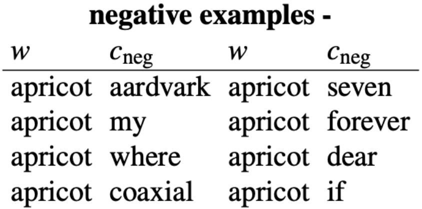

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$$p_\theta(\text{real} \mid w, c) = \frac{1}{1 + e^{-\phi_{\text{ctx}}(c)\cdot\phi_{\text{wrd}}(w)}}$$

Large dot product between w and c if they co-occur.

Negative sampling: loss function

Let $s(w, c) = \phi_{\text{ctx}}(c) \cdot \phi_{\text{wrd}}(w)$ be the score of the context and target word.

NLL loss:

$$\log(1 + e^{-s(w,c)}) + \sum_{c_n \in \mathcal{N}_{w,c}} \log(1 + e^{s(w,c_n)})$$

Let $\ell(x) = \log(1 + e^{-x})$, we have the loss for negative sampling

$$\ell(s(w, c)) + \sum_{c_n \in \mathcal{N}_{w,c}} \ell(-s(w, c_n))$$

Fasttext

Improvements over skipgram:

- Negative sampling
- Character n-gram, e.g., `<ap, app, ppl, ple, le>`

$$s(w, c) = \sum_{v \in \text{ngram}(w)} s(v, c)$$

- Faster to train and maintains similar quality to skipgram
- Well-maintained open source project: `https://fasttext.cc`

The continuous bag-of-words model

Task: given the context, predict the word in the middle

The quick brown fox jumps over the lazy dog

Similary, we can use multiclass classification for the prediction

$$p(w_i \mid w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k})$$

How to represent the context (input)?

The continuous bag-of-words model

The context is a sequence of words.

$$c = w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k}$$

$$p(w_i \mid c) = \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \phi_{\text{BoW}}(c)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \phi_{\text{BoW}}(c)\right]}$$

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- $\phi_{\text{BoW}}(c)$ sums over representations of each word in c
- Implementation is similar to the skip-gram model.

Surprising observation of word embeddings

Find similar words: top-k nearest neighbors using cosine similarity

- Size of window influences the type of similarity
- Shorter window produces syntactically similar words, e.g., Hogwarts and Sunnydale (fictional schools)
- Longer window produces topically related words, e.g., Hogwarts and Dumbledore (Harry Porter entities)

Semantic properties of word embeddings

Solve word analogy problems: a is to b as a’ is to what?

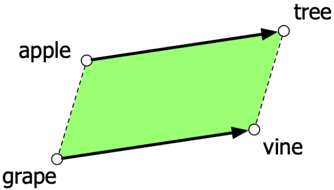

Figure: Parallelogram model (from J&H).

- man : woman :: king : queen

  $\phi_{\text{wrd}}(\text{man}) - \phi_{\text{wrd}}(\text{king}) \approx \phi_{\text{wrd}}(\text{woman}) - \phi_{\text{wrd}}(\text{queen})$
- Caveat: must exclude the three input words
- Does not work for general relations

Comparison

Count-based	Prediction-based
matrix factorization	prediction problem
fast to compute	slow (with large corpus)
interpretable components	hard to interprete but has intriguing properties

- Both uses the **distributional hypothesis**.
- Both generalize beyond text: using co-occurence between any types of objects
    - Learn product embeddings from customer orders
    - Learn region embeddings from images

Evaluate word vectors

Intrinsic evaluation

- Evaluate on the proxy task (related to the learning objective)
- Word similarity/analogy datasets (e.g., WordSim-353, SimLex-999)

Extrinsic evaluation

- Evaluate on the real/downstream task we care about
- Use word vectors as features in applications, e.g., text classification.

Summary

Key idea: formalize word representation learning as a self-supervised prediction problem

Prediction problems:

- Skip-gram: Predict context from words
- CBOW: Predict word from context
- Other possibilities:
  - Predict $\log \hat{p}(\text{word} \mid \text{context})$, e.g. GloVe
  - Contextual word embeddings (later)

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Feature learning

Linear predictor with handcrafted features: $h(x) = w \cdot \phi(x)$.

Can we learn features from data?

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Can we learn features from data?

Example:

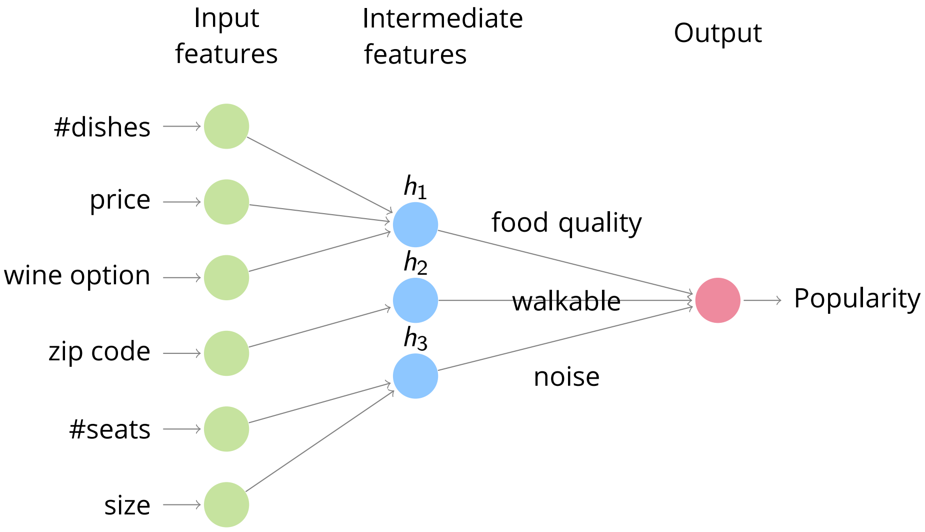
- Predict popularity of restaurants.
- Raw input: #dishes, price, wine option, zip code, #seats, size
- Decompose into subproblems:

$$h_1([\text{\#dishes, price, wine option}]) = \text{food quality}$$

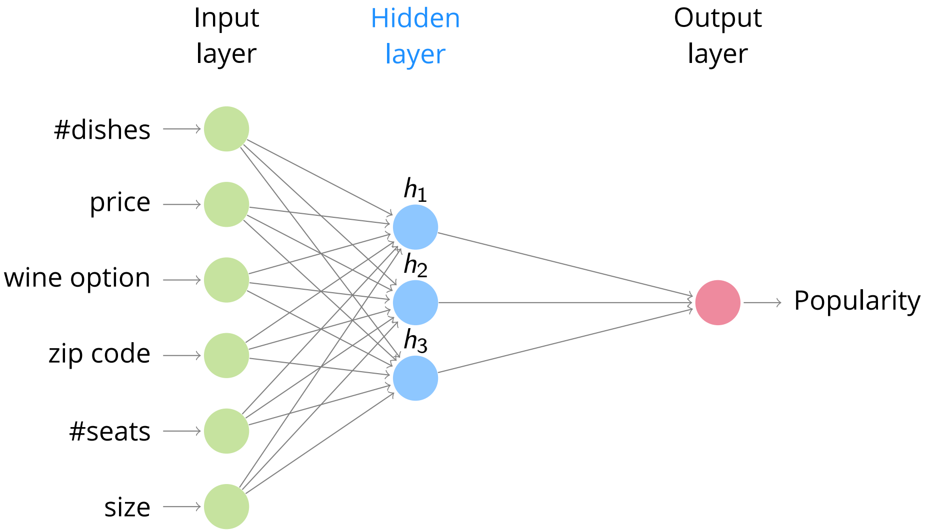
$$h_2([\text{zip code}]) = \text{walkable}$$

$$h_3([\text{\#seats, size}]) = \text{nosie}$$

Predefined subproblems



Learning intermediate features



Neural networks

Key idea: automatically learn the intermediate features.

Feature engineering: Manually specify $\phi(x)$ based on domain knowledge and learn the weights:

$$f(x) = w^T \phi(x).$$

Feature learning: Automatically learn both the features (K hidden units) and the weights:

$$h(x) = [h_1(x), \ldots, h_K(x)], \quad f(x) = w^T h(x)$$

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- How should we parametrize h_i's?

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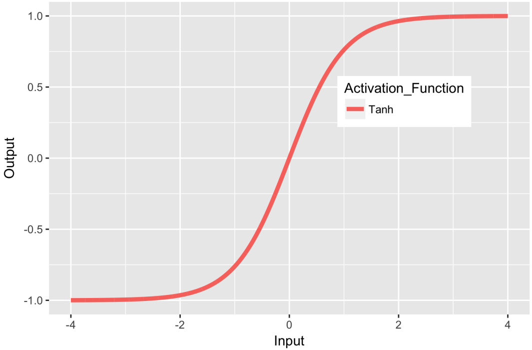
  - *Differentiable* approximations: sigmoid functions.
    - E.g., logistic function, hyperbolic tangent function, ReLU
  - Non-linearity

Activation Functions

- The **hyperbolic tangent** is a common activation function:

$$\sigma(x) = \tanh(x)\,.$$

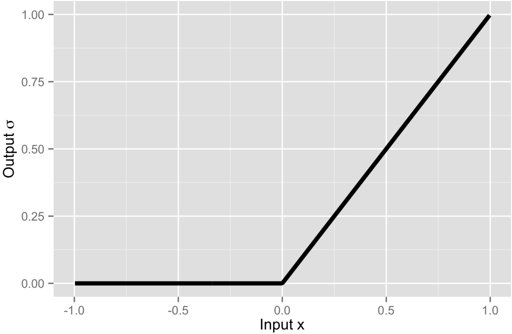


Activation Functions

- The **rectified linear** (**ReLU**) function:

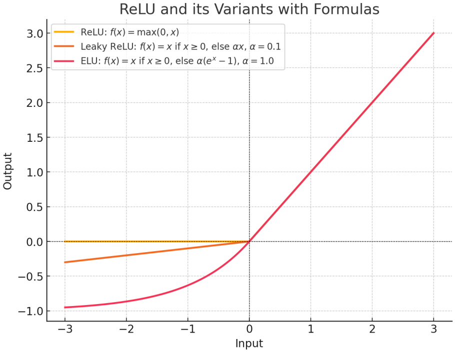
$$\sigma(x) = \max(0, x).$$

- Efficient backpropogation, sparsity, avoiding vanishing gradient
- Work well empirically.



Activation Functions

- The dying ReLU problem: neurons become inactive
- Solution: allow small gradients when the neuron is not active
  - Still need to tune the hyperparameter α

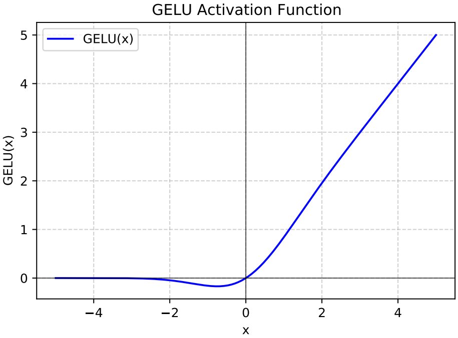


Activation Functions

- GELU: smooth transition around the origin

$$\mathrm{GELU}(x) = x \cdot \Phi(x)$$

$\Phi(x)$ is the CDF of the normal distribution



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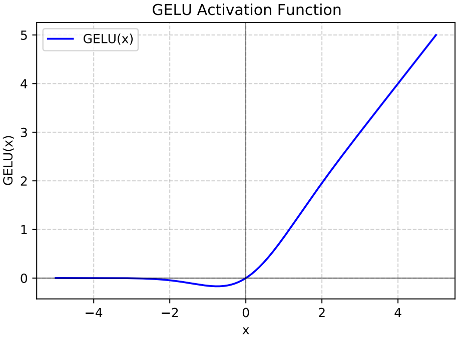
GELU Activation Function

- Downside?

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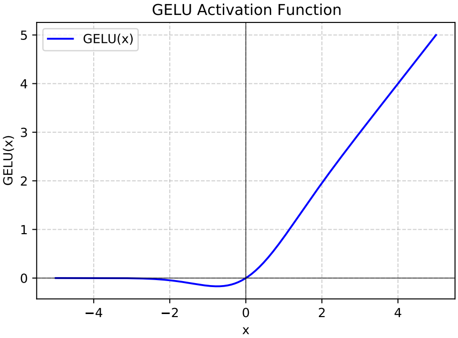
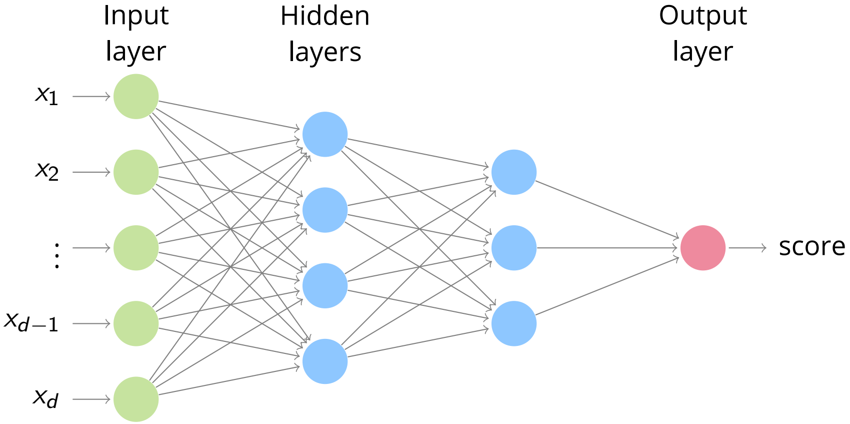


- Downside? More compute (but negligible)

Multilayer perceptron / Feed-forward neural networks

- Wider: more hidden units.
- Deeper: more hidden layers.



Multilayer Perceptron: Standard Recipe

- Each subsequent hidden layer takes the output $o \in \mathbb{R}^m$ of previous layer and produces

  $$h^{(j)}(o^{(j-1)}) = \sigma\left(W^{(j)} o^{(j-1)} + b^{(j)}\right), \text{ for } j = 2, \ldots, L$$

  where $W^{(j)} \in \mathbb{R}^{m \times m}$, $b^{(j)} \in \mathbb{R}^m$.

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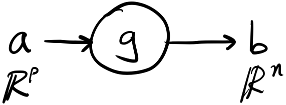
where $W^{(L+1)} \in \mathbb{R}^{k \times m}$ and $b^{(L+1)} \in \mathbb{R}^k$.
- The full neural network function is given by the *composition* of layers:
$$f(x) = \left(a \circ h^{(L)} \circ \cdots \circ h^{(1)}\right)(x) \tag{2}$$

Computation graphs

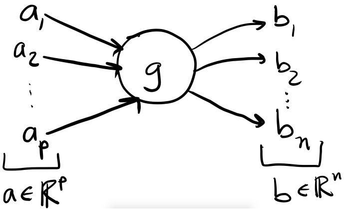
(adpated from David Rosenberg's slides)

Function as a *node* that takes in *inputs* and produces *outputs*.

- Typical computation graph:



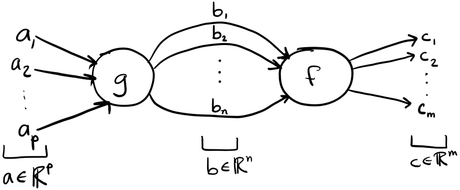
- Broken out into components:



Compose multiple functions

(adpated from David Rosenberg's slides)

Compose two functions $g : \mathbb{R}^p \to \mathbb{R}^n$ and $f : \mathbb{R}^n \to \mathbb{R}^m$: $c = f(g(a))$



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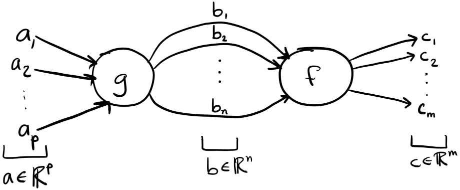

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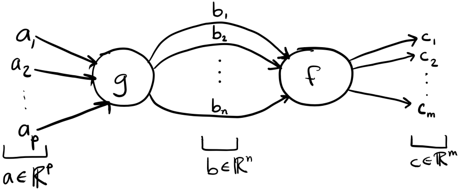

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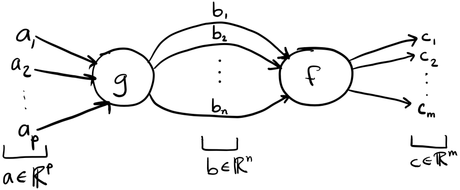

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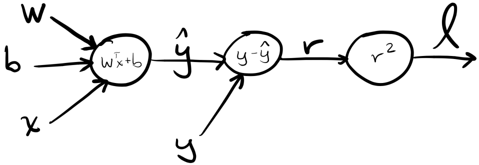
- Visualize the multivariable **chain rule**:
  - Sum changes induced on all paths from a_j to c_i.
  - Changes on one path is the product of changes across each node.

Computation graph example

(adpated from David Rosenberg's slides)



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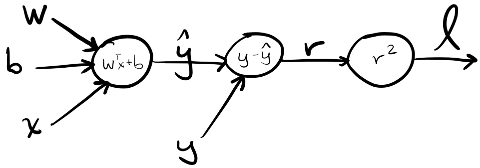


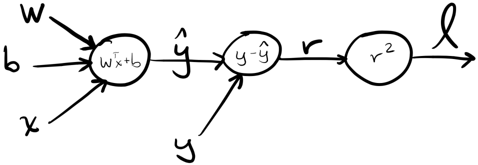
(What is this graph computing?)

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial b} = (-2r)(1) = -2r$$

$$\frac{\partial \ell}{\partial w_j} = \frac{\partial \ell}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_j} = (-2r)\, x_j = -2rx_j$$

Computation graph example

(adpated from David Rosenberg's slides)



(What is this graph computing?)

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial b} = (-2r)(1) = -2r$$

$$\frac{\partial \ell}{\partial w_j} = \frac{\partial \ell}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w_j} = (-2r)\, x_j = -2rx_j$$

Computing the derivatives in certain order allows us to save compute!

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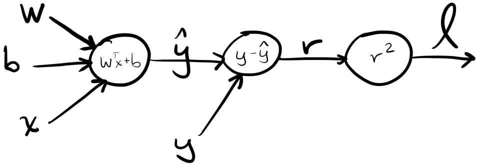

(What is this graph computing?)

$$\frac{\partial \ell}{\partial r} = 2r$$

$$\frac{\partial \ell}{\partial \hat{y}} = \frac{\partial \ell}{\partial r}\frac{\partial r}{\partial \hat{y}} = (2r)(-1) = -2r$$

$$\frac{\partial \ell}{\partial b} = \frac{\partial \ell}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial b} = (-2r)(1) = -2r$$

$$\frac{\partial \ell}{\partial w_j} = \frac{\partial \ell}{\partial \hat{y}}\frac{\partial \hat{y}}{\partial w_j} = (-2r)x_j = -2rx_j$$

Computing the derivatives in certain order allows us to save compute!

Backpropogation

Backpropogation = chain rule + dynamic programming on a computation graph

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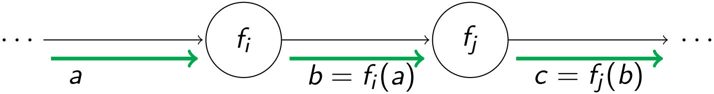
Forward pass

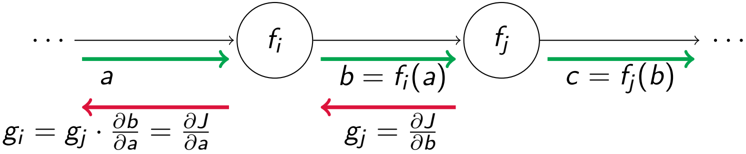
- **Topological order**: every node appears before its children
- For each node, compute the output given the input (from its parents).



Backpropogation

Backward pass

- **Reverse topological order**: every node appear after its children
- For each node, compute the partial derivative of its output w.r.t. its input, multiplied by the partial derivative from its children (chain rule).



Summary

Key idea in neural nets: feature/representation learning

Building blocks:

- Input layer: raw features (no learnable parameters)
- Hidden layer: perceptron + nonlinear activation function
- Output layer: linear (+ transformation, e.g. softmax)

Optimization:

- Optimize by SGD (implemented by back-propogation)
- Objective is non-convex, may not reach a global minimum