Distributed representation of text

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Logistics

- HW1 released. Due by next Friday.
- Plan for today:
  - Lecture: 75 minutes
  - Break: 5 minutes
  - Section by Nitish: 40 minutes

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Last week

Generative vs discriminative models for text classification

- (Multinomial) naive Bayes

What's the key assumption?

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Feature vector of text input

- BoW representation
- N-gram features (usually $n \leq 3$)

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Control the complexity of the hypothesis class

- Feature selection
- Norm regularization

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Objective

Goal: come up with a good representation of text

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- What is a good representation?
  - Leads to good task performance (often requires less training data)
  - Enables a notion of distance over text: $d(\phi(a), \phi(b))$ is small for semantically similar texts a and b

Distance functions

Euclidean distance

For $a, b \in \mathbb{R}^d$,

$$d(a, b) = \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2} \ .$$

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Cosine similarity

For $a, b \in \mathbb{R}^d$,

$$\text{sim}(a, b) = \frac{a \cdot b}{\|a\| \|b\|} = \cos \alpha$$

Angle between two vectors

Example: information retrieval

Given a set of documents and a query, use the BoW representation and cosine similarity to find the most relevant document.

What are potential problems?

Example:

Q: Who has watched Avatar?

d_1: She has watched the Wandering Earth.

d_2: Avatar was shown here last week.

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Example:

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- Similarity may be dominated by common words
- Only considers the surface form (e.g., do not account for synonyms)

TFIDF

Key idea: upweight words that carry more information about the document

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Feature map ϕ: document $\rightarrow \mathbb{R}^{|\mathcal{V}|}$

TFIDF:

$$\phi_i(d) = \underbrace{\text{count}(w_i, d)}_{\text{term frequency}} \times$$

- **Term frequency (TF)**: count of each word type in the document (same as BoW)

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- **Term frequency (TF)**: count of each word type in the document (same as BoW)
- Reweight by **inverse document frequency (IDF)**: how specific is the word type to any particular document
- Higher weight on frequent words that only occur in a few documents

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The distributional hypothesis

“You shall know a word by the company it keeps.” (Firth, 1957)

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Word guessing! (example from Eisenstein's book)

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Don't have tezgüino before you drive.

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Idea: Represent a word by its neighbors.

Step 1: Choose the context

What are the neighbors? (What type of co-occurence are we interested in?)

Example:

- word $\times$ document

	As You Like It	**Twelfth Night**	**Julius Caesar**	**Henry V**
battle	1	0	7	13
good	114	80	62	89
fool	36	58	1	4
wit	20	15	2	3

Figure 6.2 The term-document matrix for four words in four Shakespeare plays. Each cell contains the number of times the (row) word occurs in the (column) document.

Figure: Jurafsky and Martin.

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Construct a matrix where

- Row and columns represent two sets of objects
- Each entry is the (adjusted) co-occurence counts of the two objects

Step 2: Reweight counts

Upweight informative words

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	0.074	0	0.22	0.28
good	0	0	0	0
fool	0.019	0.021	0.0036	0.0083
wit	0.049	0.044	0.018	0.022

Figure 6.9 A tf-idf weighted term-document matrix for four words in four Shakespeare

Figure: Jurafsky and Martin.

Each row/column gives us a word/document representation.

Using cosine similarity, we can cluster documents, find synonyms, discover word meanings...

Pointwise mutual information

$$\mathrm{PMI}(x;y) \stackrel{\mathrm{def}}{=} \log \frac{p(x,y)}{p(x)p(y)} = \log \frac{p(x \mid y)}{p(x)} = \log \frac{p(y \mid x)}{p(y)}$$

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- Symmetric: $\mathrm{PMI}(x;y) = \mathrm{PMI}(y;x)$
- Range: $(-\infty, \min(-\log p(x), -\log p(y)))$
- Estimates:

$$\hat{p}(x \mid y) = \frac{\mathrm{count}(x,y)}{\mathrm{count}(y)} \quad \hat{p}(x) = \frac{\mathrm{count}(x)}{\sum_{x' \in \mathcal{X}} \mathrm{count}(x')}$$

- Positive PMI: $\mathrm{PPMI}(x;y) \stackrel{\text{def}}{=} \max(0, \mathrm{PMI}(x;y))$
- Application in NLP: measure association between words

Step 3: Dimensionality reduction

Motivation: want a lower-dimensional, dense representation for efficiency

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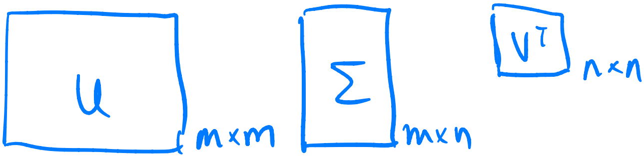
Recall **SVD**: a $m \times n$ matrix $A_{m \times n}$ (e.g., a word-document matrix), can be decomposed to

$$U_{m \times m} \Sigma_{m \times n} V^T_{n \times n} \, ,$$

where U and V are orthogonal matrices, and Σ is a diagonal matrix.



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Interpretation:

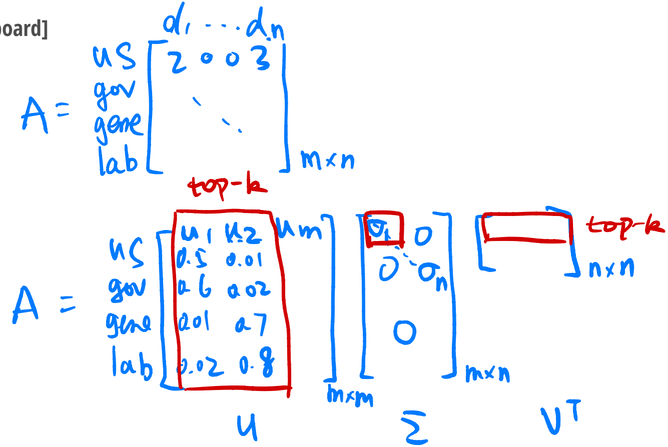
$$AA^T = (U\Sigma V^T)(V\Sigma U^T) = U\Sigma^2 U^T \; .$$

A^TA

- σ_i^2 are eigenvalues of AA^T
- Connection to PCA: If columns of A have zero mean (i.e. AA^T is the covariance matrix), then columns of U are principle components of the column space of A.

SVD for the word-document matrix

[board]



$$W = U_k \Sigma_k$$

$$(W = U_k)$$

SVD for the word-document matrix

[board]

- Run truncated SVD of the word-document matrix $A_{m \times n}$
- Each row of $U_{m \times k}\Sigma_k$ corresponds to a word vector of dimension k
- Each coordinate of the word vector corresponds to a cluster of documents (e.g., politics, music etc.)

Summary

Count-based word embeddings

1. Design the matrix, e.g. word $\times$ document, people $\times$ movie.
2. Reweight the raw counts, e.g. TFIDF, PMI.
3. Reduce dimensionality by truncated SVD.
4. Use word/person/etc. vectors in downstream tasks.

Key idea:

- Represent an object by its connection to other objects.
- For NLP, the word meaning can be represented by the context it occurs in.
- Infer latent features using co-occurence statistics

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Learning word embeddings

Goal: map each word to a vector in $\mathbb{R}^d$ such that *similar* words also have *similar* word vectors.

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Can we formalize this as a prediction problem?

- Needs to be self-supervised since our data is unlabeled.

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Intuition: Similar words occur in similar contexts

- Predict the context given a word f : word $\rightarrow$ context
- Words that tend to occur in same contexts will have similar representation

The skip-gram model

Task: given a word, predict its neighboring words within a window

The quick brown fox jumps over the lazy dog

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Assume conditional independence of the context words:

$$p(w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k} \mid w_i) = \prod_{j=i-k, j \neq i}^{i+k} p(w_j \mid w_i)$$

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How to model $p(w_j \mid w_i)$? Multiclass classification

The skip-gram model

Use the softmax function to predict context words from the center word

$$p(w_j \mid w_i) = \frac{\exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}$$

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class $\quad \theta_j \cdot \phi(w_i)$

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🤔 What's the difference from multinomial logistic regression?

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What's the difference from multinomial logistic regression?

Implementation:

- Matrix form: $\phi\colon\ w \mapsto A_{d \times |\mathcal{V}|}\phi_{\text{one-hot}}(w)$, ϕ can be implemented as a dictionary

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Implementation:

- Matrix form: $\phi : w \mapsto A_{d \times |\mathcal{V}|} \phi_{\text{one-hot}}(w)$, ϕ can be implemented as a dictionary
- Learn parameters by MLE and SGD (Is the objective convex?)
- ϕ_{wrd} is taken as the word embedding

Negative sampling

Challenge in MLE: computing the normalizer is expensive (try calculate the gradient)!

$$\ell(\theta) = \sum_{(c,w)\in D} \log p(c \mid w)$$

$$= \sum_{(c,w)\in D} \phi_c(c)\cdot\phi_w(w) - \log \sum_{w'\in V} \phi_c(c)\cdot\phi_w(w')$$

Negative sampling

Challenge in MLE: computing the normalizer is expensive (try calculate the gradient)!

Key idea: solve a binary classification problem instead

Is the (word, context) pair real or fake?

positive examples +

w	c_{pos}
apricot	tablespoon
apricot	of
apricot	jam
apricot	a

negative examples -

w	c_{neg}	w	c_{neg}
apricot	aardvark	apricot	seven
apricot	my	apricot	forever
apricot	where	apricot	dear
apricot	coaxial	apricot	if

$$p_\theta(\text{real} \mid w, c) = \frac{1}{1 + e^{-\phi_{\text{ctx}}(c)\cdot\phi_{\text{wrd}}(w)}}$$

The continuous bag-of-words model

Task: given the context, predict the word in the middle

The quick brown fox jumps over the lazy dog

Similary, we can use logistic regression for the prediction

$$p(w_i \mid w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k})$$

How to represent the context (input)?

The continuous bag-of-words model

The context is a sequence of words.

$$c = w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k}$$

$$p(w_i \mid c) = \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \phi_{\text{BoW}}(c)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \phi_{\text{BoW}}(c)\right]}$$

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$$= \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \sum_{w' \in c} \phi_{\text{ctx}}(w')\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \sum_{w' \in c} \phi_{\text{ctx}}(w')\right]}$$

- $\phi_{\text{BoW}}(c)$ sums over representations of each word in c
- Implementation is similar to the skip-gram model.

Semantic properties of word embeddings

Find similar words: top-k nearest neighbors using cosine similarity

- Size of window influences the type of similarity
- Shorter window produces syntactically similar words, e.g., Hogwarts and Sunnydale (fictional schools)
- Longer window produces topically related words, e.g., Hogwarts and Dumbledore (Harry Porter entities)

Semantic properties of word embeddings

Solve word analogy problems: a is to b as a' is to what?

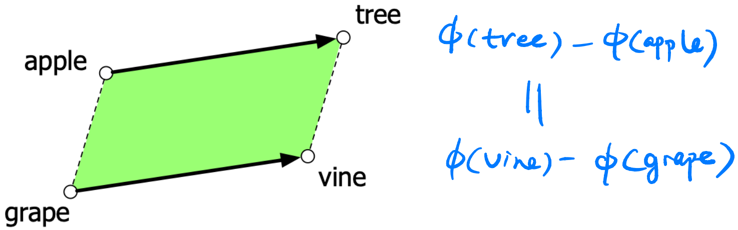

Figure: Parallelogram model (from J&H).

- man : woman :: king : queen

  $\phi_{\text{wrd}}(\text{man}) - \phi_{\text{wrd}}(\text{king}) \approx \phi_{\text{wrd}}(\text{woman}) - \phi_{\text{wrd}}(\text{queen})$
- Caveat: must exclude the three input words
- Does not work for general relations

Comparison

Count-based	Prediction-based
matrix factorization	prediction problem
fast to compute	slow (with large corpus) but more flexible
interpretable components	hard to interprete but has intriguing properties

- Both uses the **distributional hypothesis**.
- Both generalize beyond text: using co-occurence between any types of objects
  - Learn product embeddings from customer orders
  - Learn region embeddings from images

Evaluate word vectors

Intrinsic evaluation

- Evaluate on the proxy task (related to the learning objective)
- Word similarity/analogy datasets (e.g., WordSim-353, SimLex-999)

Extrinsic evaluation

- Evaluate on the real/downstream task we care about
- Use word vectors as features in NER, parsing etc.

Summary

Key idea: formalize word representation learning as a self-supervised prediction problem

Prediction problems:

- Skip-gram: Predict context from words
- CBOW: Predict word from context
- Other possibilities:
  - Predict $\log \hat{p}(\text{word} \mid \text{context})$, e.g. GloVe
  - Contextual word embeddings (later)

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Feature learning

Linear predictor with handcrafted features: $f(x) = w \cdot \phi(x)$.

Can we learn intermediate features?

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Can we learn intermediate features?

Example:

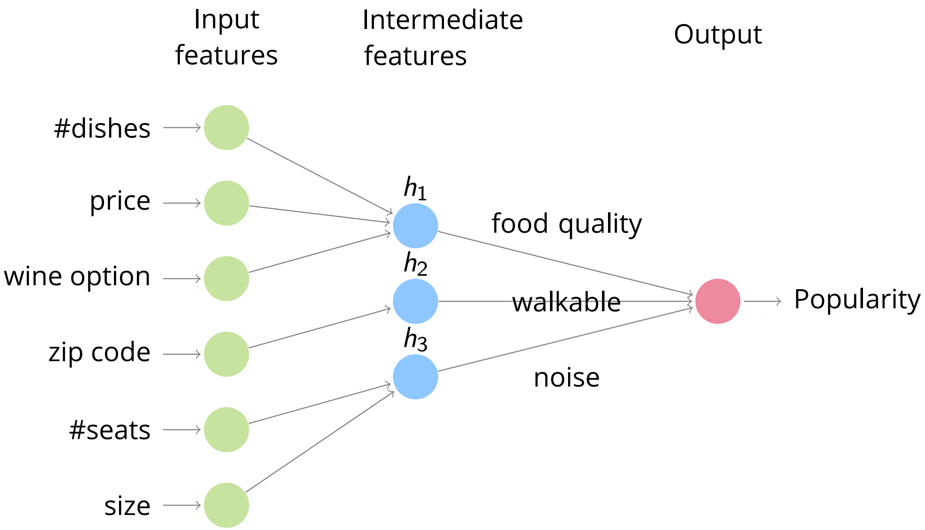
- Predict popularity of restaurants.
- Raw input: #dishes, price, wine option, zip code, #seats, size
- Decompose into subproblems:

$$h_1([\text{\#dishes, price, wine option}]) = \text{food quality}$$

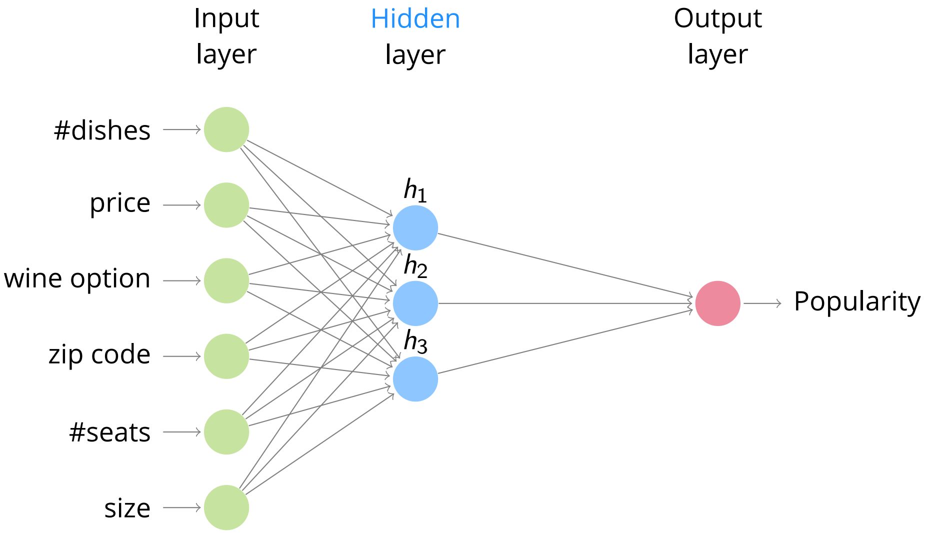
$$h_2([\text{zip code}]) = \text{walkable}$$

$$h_3([\text{\#seats, size}]) = \text{nosie}$$

Predefined subproblems



Learning intermediate features



Neural networks

Key idea: automatically learn the intermediate features.

Feature engineering: Manually specify $\phi(x)$ based on domain knowledge and learn the weights:

$$f(x) = w^T \phi(x).$$

Feature learning: Automatically learn both the features (K hidden units) and the weights:

$$h(x) = [h_1(x), \ldots, h_K(x)], \quad f(x) = w^T h(x)$$

Activation function

- How should we parametrize h_i's? Can it be linear?

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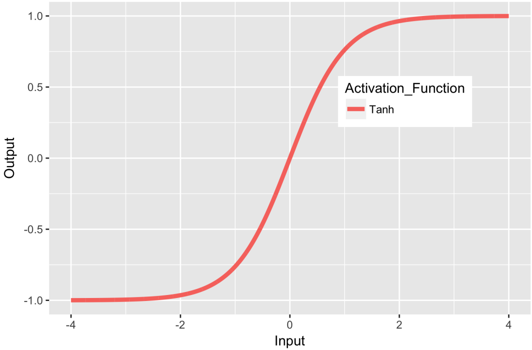
- Two-layer neural network (one hidden layer and one output layer) with K hidden units:

$$f(x) = \sum_{k=1}^{K} w_k h_k(x) = \sum_{k=1}^{K} w_k \sigma({v_k}^T x) \tag{2}$$

Activation Functions

- The **hyperbolic tangent** is a common activation function:

$$\sigma(x) = \tanh(x) \,.$$

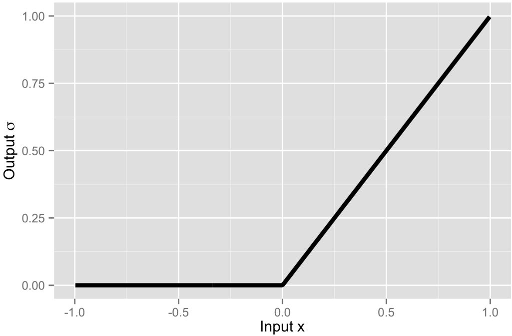


Activation Functions

- More recently, the **rectified linear** (**ReLU**) function has been very popular:

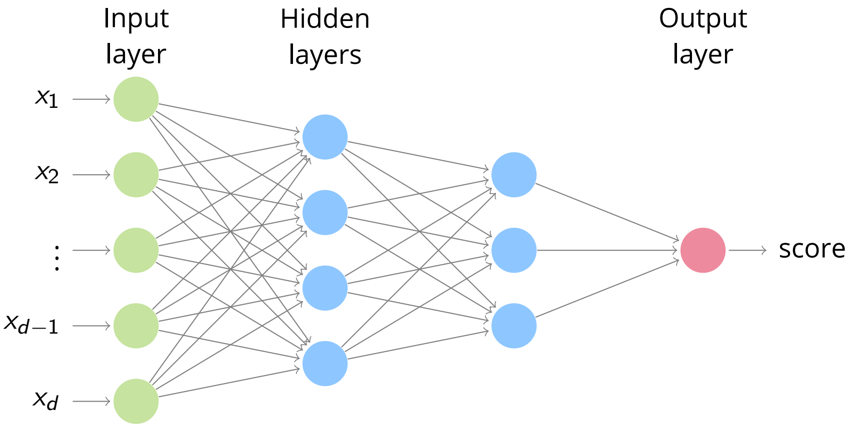
$$\sigma(x) = \max(0, x).$$

- Much faster to calculate, and to calculate its derivatives.
- Work well empirically.



Multilayer perceptron / Feed-forward neural networks

- Wider: more hidden units.
- Deeper: more hidden layers.



Multilayer Perceptron: Standard Recipe

- Each subsequent hidden layer takes the output $o \in \mathbb{R}^m$ of previous layer and produces

$$h^{(j)}(o^{(j-1)}) = \sigma\left(W^{(j)}o^{(j-1)} + b^{(j)}\right), \text{ for } j = 2, \ldots, L$$

where $W^{(j)} \in \mathbb{R}^{m \times m}$, $b^{(j)} \in \mathbb{R}^m$.

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- Last layer typically gives us a score. (How to do classification?)