



# Operations on Word Vectors

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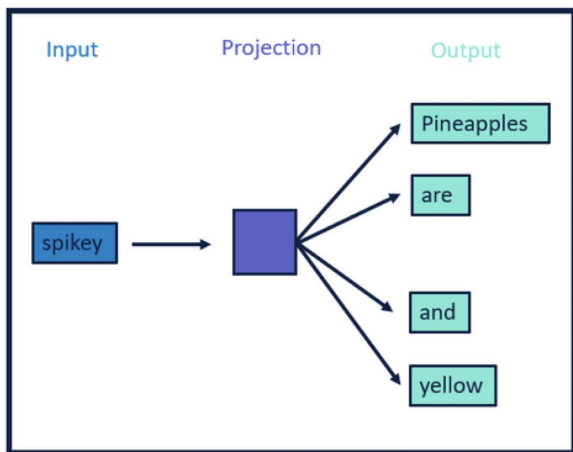
# Recap

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**Skip-gram model:** Given a word, predict its neighbouring words within a window.



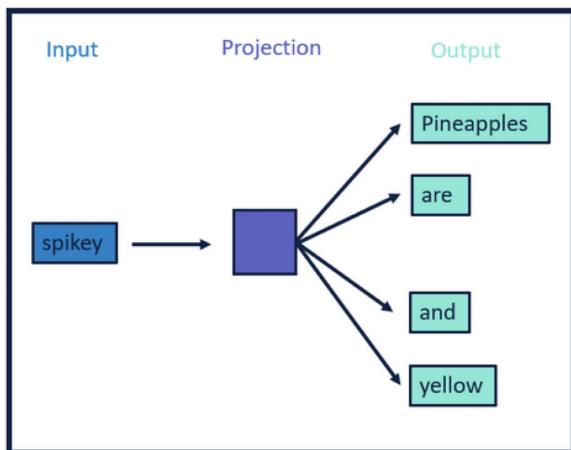
Skip-gram

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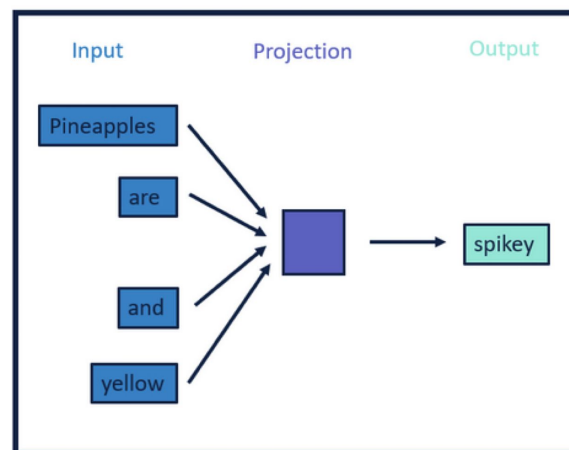
**Goal:** Map each word to a vector in  $\mathbb{R}^d$  such that similar words have similar vectors.

**Skip-gram model:** Given a word, predict its neighbouring words within a window.

**Continuous bag-of-words model:** Given the context, predict the missing word.



Skip-gram



CBOW

# Recap

**GloVe:** Global Vectors (Pennington et al., 2014) — Use co-occurrence matrix of each word pair.  $N(w, c)$  is the co-occurrence count between word  $w$  and context  $c$ .

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$$J(\theta) = \sum_{w,c \in V} f(N(w, c)) \cdot (u_c^T v_w + b_c + \bar{b}_w - \log N(w, c))^2$$

Diagram illustrating the components of the GloVe loss function:

- context vector (points to  $u_c^T$ )
- word vector (points to  $v_w$ )
- bias terms (also learned) (points to  $b_c$  and  $\bar{b}_w$ )

The function  $f(N(w, c))$  is also indicated by a bracket and arrow pointing downwards.

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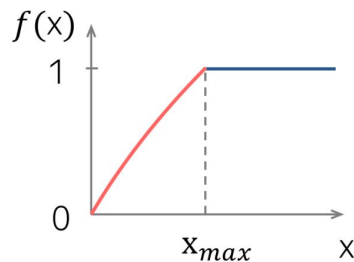
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$$J(\theta) = \sum_{w,c \in V} \underbrace{f(N(w, c))}_{\text{weighting function}} \cdot (u_c^T v_w + b_c + \overline{b_w} - \log N(w, c))^2$$

Diagram labels above the equation:  
context vector (points to  $u_c$ )  
word vector (points to  $v_w$ )  
bias terms (also learned) (points to  $b_c$  and  $\overline{b_w}$ )

Weighting function to:

- penalize rare events
- not to over-weight frequent events



$$\begin{cases} (x/x_{max})^\alpha & \text{if } x < x_{max}, \\ 1 & \text{otherwise.} \end{cases}$$

$$\alpha = 0.75, x_{max} = 100$$

# Similarity between Word Vectors

**Question:** Do the learnt word embeddings satisfy the desired property of similarity?

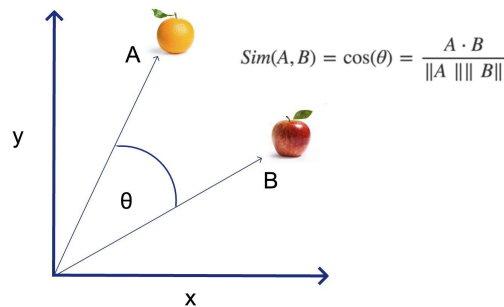


# Similarity between Word Vectors

**Question:** Do the learnt word embeddings satisfy the desired property of similarity?

Use cosine similarity between any two word vectors.

## Cosine Similarity



# Word Analogy Task

In word analogy tasks, we ask questions like “a is to b as c is to \_\_\_”

Example: “London is to UK as Amsterdam is to Netherlands”

# Word Analogy Task

For  $a \rightarrow b :: c \rightarrow ?$ , given word vectors  $v_a$ ,  $v_b$  and  $v_c$ , we will find a word  $d$  such that  $v_a - v_b \sim v_c - v_d$ .

# Word Analogy Task

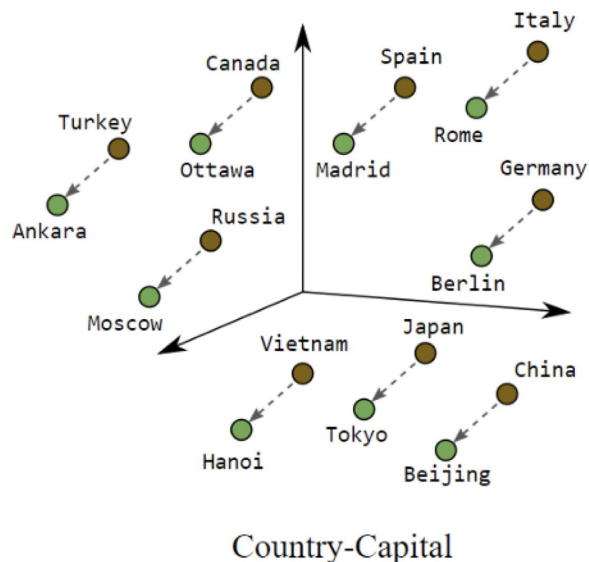
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# Word Similarity Tasks

- WordSim353
- SimLex-999 (similarity rather than relatedness)

<b>Pair</b>	<b>Simlex-999 rating</b>	<b>WordSim-353 rating</b>
<i>coast - shore</i>	9.00	9.10
<i>clothes - closet</i>	1.96	8.00

# Bias in Word Vectors

The difference  $v_a - v_b$  represents the ‘concept’ — if  $a$  is woman and  $b$  is man, then it represents ‘gender’.

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Compute projections of occupations on this difference  $v_a - v_b$

## Extreme *she* occupations

- |                 |                       |                        |
|-----------------|-----------------------|------------------------|
| 1. homemaker    | 2. nurse              | 3. receptionist        |
| 4. librarian    | 5. socialite          | 6. hairdresser         |
| 7. nanny        | 8. bookkeeper         | 9. stylist             |
| 10. housekeeper | 11. interior designer | 12. guidance counselor |

## Extreme *he* occupations

- |                |                   |                |
|----------------|-------------------|----------------|
| 1. maestro     | 2. skipper        | 3. protege     |
| 4. philosopher | 5. captain        | 6. architect   |
| 7. financier   | 8. warrior        | 9. broadcaster |
| 10. magician   | 11. fighter pilot | 12. boss       |



# Bias in Word Vectors

Similarly, we can obtain vectors for other concepts like race and religion.

Compute projections of occupations on this difference  $v_a - v_b$ .

<b>Racially Biased Analogies</b>	
black → criminal	caucasian → police
asian → doctor	caucasian → dad
caucasian → leader	black → led
<b>Religiously Biased Analogies</b>	
muslim → terrorist	christian → civilians
jewish → philanthropist	christian → stooge
christian → unemployed	jewish → pensioners

Note: The vectors were obtained from training on reddit data from USA users

# Debiasing Word Vectors

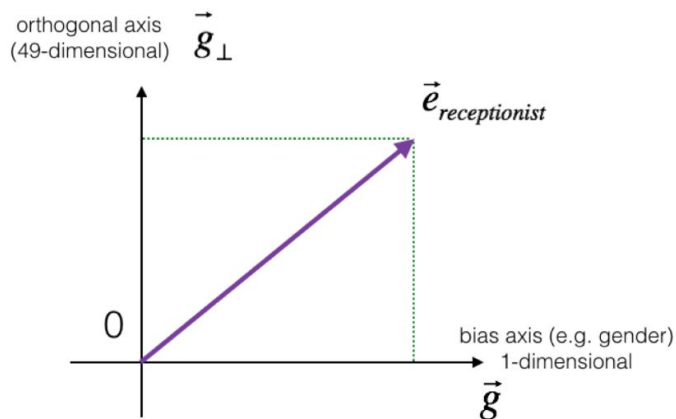
For a concept vector  $g$  and word vector  $e$ , obtain the biased component:

$$e_{\text{biased}} = \frac{e \cdot g}{\|g\|^2} g$$

Subtract from the original vector to obtain the debiased vector

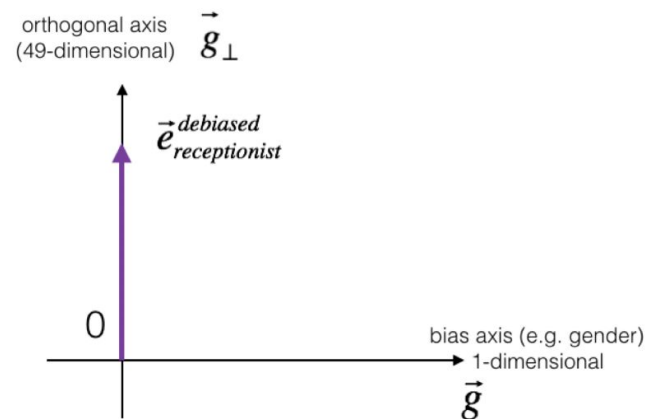
$$e_{\text{debiased}} = e - e_{\text{biased}}$$

# Debiasing Word Vectors



**before neutralizing.**

“receptionist” is positively correlated with the bias axis



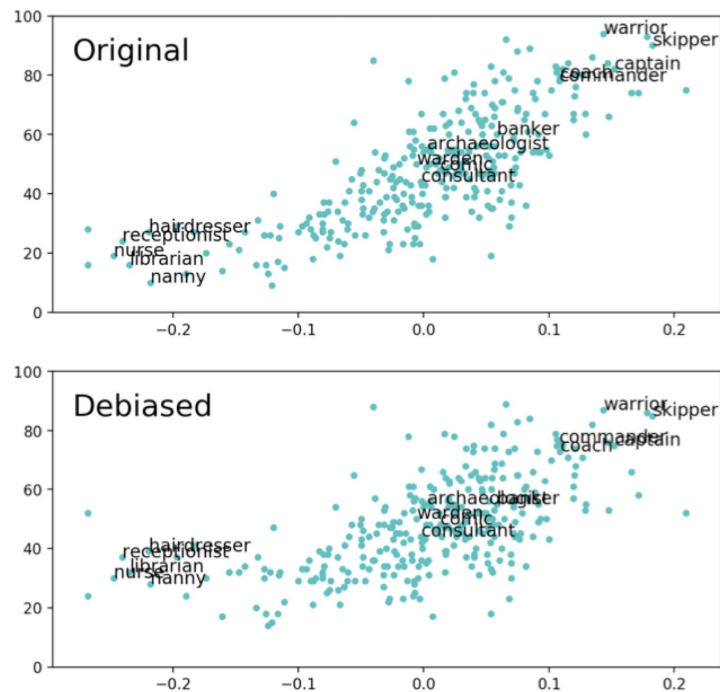
**after neutralizing.**

debiased version, with the component in the direction of the bias axis ( $g$ ) zeroed out

# Debiasing Word Vectors

Previous method ensures that vector is orthogonal to the concept vector.

Not always effective in debiasing — the word vectors corresponding to occupations are still clustered according to gender.



# Other Debiasing Methods

Ravfogel et al 2020:

- There is no single direction corresponding to concepts — it can span in multiple directions.
- Propose Iterative Null-space Projection (INLP) — iteratively neutralise/debias the vectors.

# Other Debiasing Methods

## **Algorithm 1** Iterative Nullspace Projection (INLP)

**Input:**  $(X, Z)$ : a training set of vectors and protected attributes  
n: Number of rounds

**Result:** A projection matrix  $P$

**Function** `GetProjectionMatrix` ( $X, Z$ ):

```
 $X_{projected} \leftarrow X$   
 $P \leftarrow I$   
for  $i \leftarrow 1$  to  $n$  do  
   $W_i \leftarrow \text{TrainClassifier}(X_{projected}, Z)$   
   $B_i \leftarrow \text{GetNullSpaceBasis}(W_i)$   
   $P_{N(W_i)} \leftarrow B_i B_i^T$   
   $P \leftarrow P_{N(W_i)} P$   
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e.g. Dataset of (occupation, gender) where we have word vectors for each occupation along with the biased gender.

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→ e.g. Train a linear classifier to predict gender from occupation.

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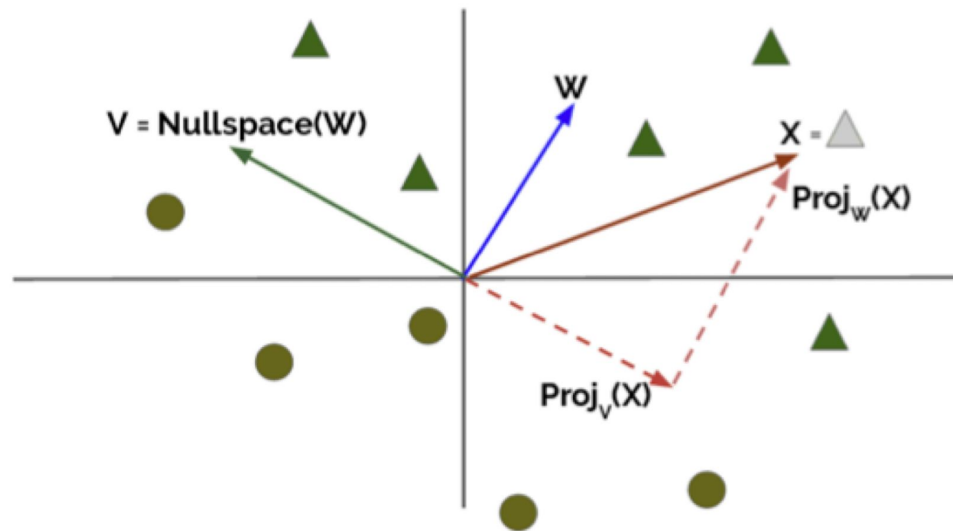
```
return  $P$ 
```

Project  $X$  onto nullspace of  $W \rightarrow$   
predicting  $Z$  (e.g. gender) from  
new  $X$  will not work.



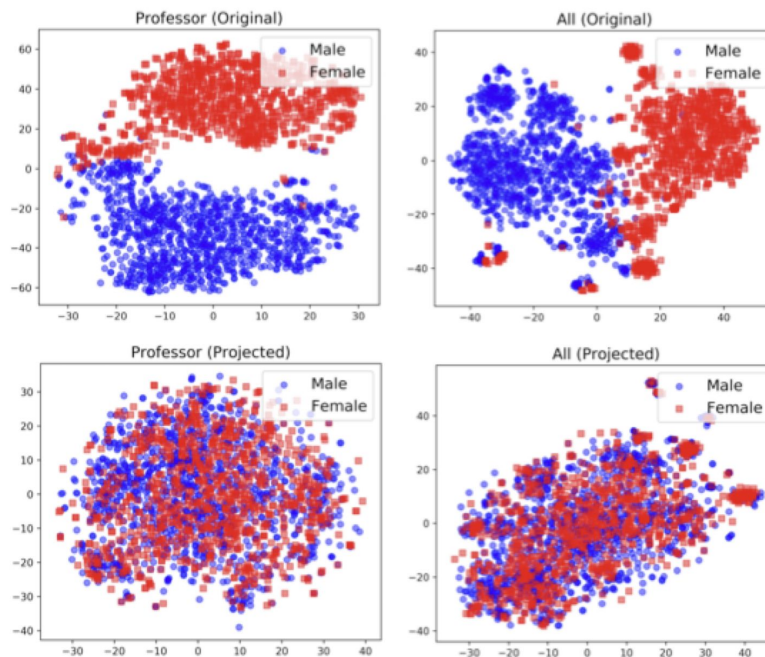
# Other Debiasing Methods

- $W$ : weight of a linear classifier trained to predict  $Z$  from  $X$
- Project on null-space
- Iterate



# Other Debiasing Methods

- Does not suffer from the issue we saw with earlier debiasing method.
- Representations are now not clustered according to protected attribute (e.g. gender).



# Summary

- Word vectors encode a notion of similarity, which can be helpful for retrieval, word analogy tasks etc.
- Word vectors can encode biases from the data → Need to evaluate and use appropriate debiasing methods.

# Acknowledgement

This presentation is adapted from Nitish Joshi, 31st January 2022