# Distributed representation of text

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# Logistics

- HW1 released. Due by next Friday 12pm.
- Plan for today:
  - Wrap up logistic regression from last week
  - Word embeddings
  - Neural networks basics

# Table of Contents

# Last week

## Generative vs discriminative models

for text classification

- (Multinomial) naive Bayes

What’s the key assumption?

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**Feature vector** of text input

- BoW representation
- N-gram features (usually $n \leq 3$)

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- What is a good representation?
  - Leads to good task performance (often requires less training data)
  - Enables a notion of distance over text: $d(\phi(a), \phi(b))$ is small for semantically similar texts $a$ and $b$

# Distance functions

**Euclidean distance**

For $a, b \in \mathbb{R}^d$,

$$d(a, b) = \sqrt{\sum_{i=1}^{d} (a_i - b_i)^2} .$$

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**Cosine similarity**

For $a, b \in \mathbb{R}^d$,

$$\text{sim}(a, b) = \frac{a \cdot b}{\|a\| \|b\|} = \cos \alpha$$

Angle between two vectors

# Example: information retrieval

Given a set of documents and a query, use the BoW representation and cosine similarity to find the most relevant document.

What are potential problems?

Example:

Q: Who has watched Barbie?

She has watched Oppenheimer.

Barbie was shown here last week.

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- Similarity may be dominated by common words
- Only considers the surface form (e.g., do not account for synonyms)

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Construct a feature map $\phi$: document $\rightarrow \mathbb{R}^{|\mathcal{V}|}$

TFIDF:

$$\phi_w(d) = \underbrace{\text{count}(w, d)}_{\text{tf}(w,d)} \times$$

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- **Term frequency (TF)**: count of each word type in the document (same as BoW)
- Reweight by **inverse document frequency (IDF)**: how specific is the word to any particular document
- Higher weight on frequent words that only occur in a few documents

# TFIDF example

| | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---|---|---|---|---|
| **battle** | 0.074 | 0 | 0.22 | 0.28 |
| **good** | 0 | 0 | 0 | 0 |
| **fool** | 0.019 | 0.021 | 0.0036 | 0.0083 |
| **wit** | 0.049 | 0.044 | 0.018 | 0.022 |

**Figure 6.9** A tf-idf weighted term-document matrix for four words in four Shakespeare plays, using the counts in Fig. 6.2. For example the 0.049 value for *wit* in *As You Like It* is the product of $\text{tf} = \log_{10}(20+1) = 1.322$ and $\text{idf} = .037$. Note that the idf weighting has eliminated the importance of the ubiquitous word *good* and vastly reduced the impact of the almost-ubiquitous word *fool*.

Figure: From Jurafsky and Martin.

Why do some words have zero weights?

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Idea: Represent a word by its neighbors.

# Step 1: Choose the context

What are the neighbors? (What type of co-occurence are we interested in?)

Example:

- word $\times$ document

| | As You Like It | Twelfth Night | Julius Caesar | Henry V |
|---|---|---|---|---|
| **battle** | 1 | 0 | 7 | 13 |
| **good** | 114 | 80 | 62 | 89 |
| **fool** | 36 | 58 | 1 | 4 |
| **wit** | 20 | 15 | 2 | 3 |

**Figure 6.2** The term-document matrix for four words in four Shakespeare plays. Each cell contains the number of times the (row) word occurs in the (column) document.

Figure: Jurafsky and Martin.

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Construct a matrix where

- Row and columns represent two sets of objects
- Each entry is the (adjusted) co-occurence counts of the two objects

## Step 2: Reweight counts

Upweight informative words

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Using cosine similarity, we can cluster documents, find synonyms, discover word meanings...

# An alternative way to reweighting using pointwise mutual information

$$\mathrm{PMI}(x; y) \stackrel{\text{def}}{=} \log \frac{p(x, y)}{p(x)p(y)} = \log \frac{p(x \mid y)}{p(x)} = \log \frac{p(y \mid x)}{p(y)}$$

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- Symmetric: $\mathrm{PMI}(x; y) = \mathrm{PMI}(y; x)$
- Estimates:

$$\hat{p}(x \mid y) = \frac{\mathrm{count}(x, y)}{\mathrm{count}(y)} \quad \text{how often do word } x \text{ occur in the neighborhood of } y$$

$$\hat{p}(x) = \frac{\mathrm{count}(x)}{\sum_{x' \in \mathcal{X}} \mathrm{count}(x')} \quad \text{how often do word } x \text{ occur in the corpus}$$

# Positive PMI / PPMI

- Range: $(-\infty, \min(-\log p(x), -\log p(y)))$
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# Positive PMI / PPMI

- Range: $(-\infty, \min(-\log p(x), -\log p(y)))$
- What does negative PMI mean?
  - Two words co-occur less frequently than chance
  - Need large data to estimate small probabilities
  - Difficult to judge unrelatedness by humans
- **Positive PMI**: $\text{PPMI}(x; y) \stackrel{\text{def}}{=} \max(0, \text{PMI}(x; y))$
- Application in NLP: measure association between words

## Step 3: Dimensionality reduction

Motivation: want a lower-dimensional, dense representation for efficiency

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Recall **SVD**: a $m \times n$ matrix $A_{m \times n}$ (e.g., a word-document matrix), can be decomposed to

$$U_{m \times m} \Sigma_{m \times n} V_{n \times n}^T \ ,$$

where $U$ and $V$ are orthogonal matrices, and $\Sigma$ is a diagonal matrix.

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where $U$ and $V$ are orthogonal matrices, and $\Sigma$ is a diagonal matrix.

Interpretation:

$$AA^T = (U \Sigma V^T)(V \Sigma U^T) = U \Sigma^2 U^T .$$

- $\sigma_i^2$ are eigenvalues of $AA^T$
- Connection to PCA: If columns of $A$ have zero mean (i.e. $AA^T$ is the covariance matrix), then columns of $U$ are principle components of the column space of $A$.

# SVD for the word-document matrix

**[board]**

# SVD for the word-document matrix

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Computing the dense word vectors:

- Run truncated SVD of the word-document matrix $A_{m \times n}$
- Each row of $U_{m \times k}\Sigma_k$ corresponds to a word vector of dimension $k$
- Each coordinate of the word vector corresponds to a cluster of documents (e.g., politics, music etc.)

# Summary

**Count-based word embeddings**

1. Design the matrix, e.g. word $\times$ document, people $\times$ movie.
2. Reweight the raw counts, e.g. TFIDF, PPMI.
3. Reduce dimensionality by truncated SVD.
4. Use word/person/etc. vectors in downstream tasks.

**Key idea**:

- Intuition: Represent an object by its connection to other objects.
- Lexical semantics: the word meaning can be represented by the context it occurs in.
- Linear algebra: Infer clusters (e.g., concepts, topics) using co-occurence statistics

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# Learning word embeddings

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**Distributional hypothesis**: Similar words occur in similar contexts

- Predict the context given a word $f : \text{word} \rightarrow \text{context}$
- Words that tend to occur in same contexts will have similar representation

# The skip-gram model

**Task**: given a word, predict its neighboring words within a window

The quick brown fox jumps over the lazy dog

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Assume **conditional independence** of the context words:

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How to model $p(w_j \mid w_i)$? Multiclass classification

# The skip-gram model

Use the softmax function to predict context words from the center word

$$p(w_j \mid w_i) = \frac{\exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{ctx}}(w_j) \cdot \phi_{\text{wrd}}(w_i)\right]}$$

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Implementation:

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- Learn parameters by MLE and SGD (Is the objective convex?)

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Implementation:

- $\phi \colon w \to \mathbb{R}^d$. For each word $w$, learn two vectors. $\phi$ can be implemented as a dictionary
- Learn parameters by MLE and SGD (Is the objective convex?)
- $\phi_{\text{wrd}}$ is taken as the word embedding

# Negative sampling

Challenge in MLE: computing the normalizer is expensive (try calculate the gradient)!

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Challenge in MLE: computing the normalizer is expensive (try calculate the gradient)!

Key idea: solve a binary classification problem instead

Is the (word, context) pair real or fake?

**positive examples +**

| $w$ | $c_{\text{pos}}$ |
|---|---|
| apricot | tablespoon |
| apricot | of |
| apricot | jam |
| apricot | a |

**negative examples -**

| $w$ | $c_{\text{neg}}$ | $w$ | $c_{\text{neg}}$ |
|---|---|---|---|
| apricot | aardvark | apricot | seven |
| apricot | my | apricot | forever |
| apricot | where | apricot | dear |
| apricot | coaxial | apricot | if |

$$p_\theta(\text{real} \mid w, c) = \frac{1}{1 + e^{-\phi_{\text{ctx}}(c)\cdot\phi_{\text{wrd}}(w)}}$$

# The continuous bag-of-words model

Task: given the context, predict the word in the middle

The quick brown fox jumps over the lazy dog

Similary, we can use logistic regression for the prediction

$$p(w_i \mid w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k})$$

How to represent the context (input)?

# The continuous bag-of-words model

The context is a sequence of words.

$$c = w_{i-k}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+k}$$

$$p(w_i \mid c) = \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \phi_{\text{BoW}}(c)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \phi_{\text{BoW}}(c)\right]}$$

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$$\begin{aligned} p(w_i \mid c) &= \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \phi_{\text{BoW}}(c)\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \phi_{\text{BoW}}(c)\right]} \\ &= \frac{\exp\left[\phi_{\text{wrd}}(w_i) \cdot \sum_{w' \in c} \phi_{\text{ctx}}(w')\right]}{\sum_{w \in \mathcal{V}} \exp\left[\phi_{\text{wrd}}(w) \cdot \sum_{w' \in c} \phi_{\text{ctx}}(w')\right]} \end{aligned}$$

- $\phi_{\text{BoW}}(c)$ sums over representations of each word in $c$
- Implementation is similar to the skip-gram model.

# Semantic properties of word embeddings

Find similar words: top-$k$ nearest neighbors using cosine similarity

- Size of window influences the type of similarity
- Shorter window produces syntactically similar words, e.g., Hogwarts and Sunnydale (fictional schools)
- Longer window produces topically related words, e.g., Hogwarts and Dumbledore (Harry Porter entities)

# Semantic properties of word embeddings

Solve word analogy problems: a is to b as a’ is to what?

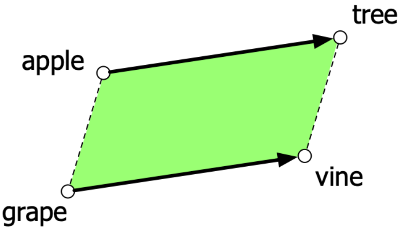


Figure: Parallelogram model (from J&H).

- man : woman :: king : queen

  $\phi_{\text{wrd}}(\text{man}) - \phi_{\text{wrd}}(\text{king}) \approx \phi_{\text{wrd}}(\text{woman}) - \phi_{\text{wrd}}(\text{queen})$
- Caveat: must exclude the three input words
- Does not work for general relations

# Comparison

| Count-based | Prediction-based |
|---|---|
| matrix factorization | prediction problem |
| fast to compute | slow (with large corpus) but more flexible |
| interpretable components | hard to interprete but has intriguing properties |

- Both uses the **distributional hypothesis**.
- Both generalize beyond text: using co-occurence between any types of objects
  - Learn product embeddings from customer orders
  - Learn region embeddings from images

# Evaluate word vectors

**Intrinsic evaluation**

- Evaluate on the proxy task (related to the learning objective)
- Word similarity/analogy datasets (e.g., WordSim-353, SimLex-999)

**Extrinsic evaluation**

- Evaluate on the real/downstream task we care about
- Use word vectors as features in NER, parsing etc.

# Summary

Key idea: formalize word representation learning as a self-supervised prediction problem

Prediction problems:

- Skip-gram: Predict context from words
- CBOW: Predict word from context
- Other possibilities:
  - Predict $\log \hat{p}(\text{word} \mid \text{context})$, e.g. GloVe
  - Contextual word embeddings (later)

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# Feature learning

Linear predictor with handcrafted features: $f(x) = w \cdot \phi(x)$.

Can we learn intermediate features?

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Example:

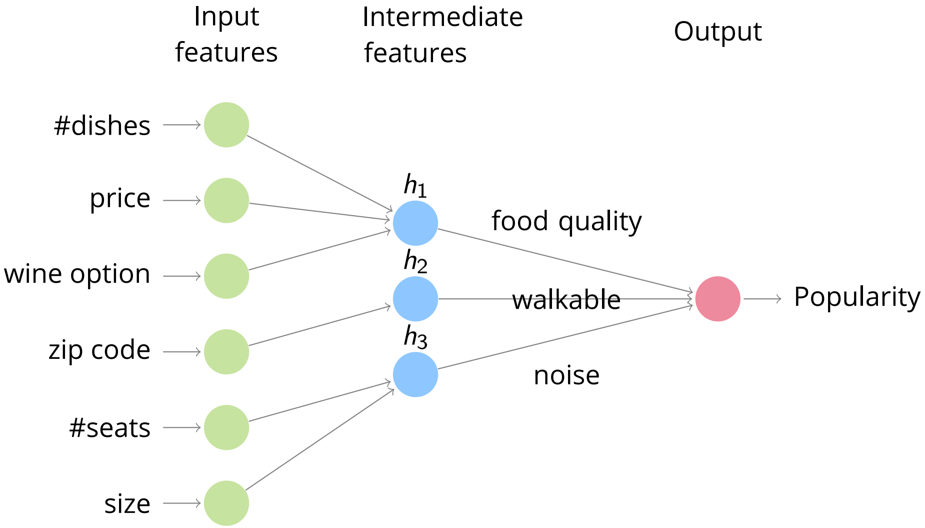
- Predict popularity of restaurants.
- Raw input: #dishes, price, wine option, zip code, #seats, size
- Decompose into subproblems:

$$h_1(\text{[\#dishes, price, wine option]}) = \text{food quality}$$

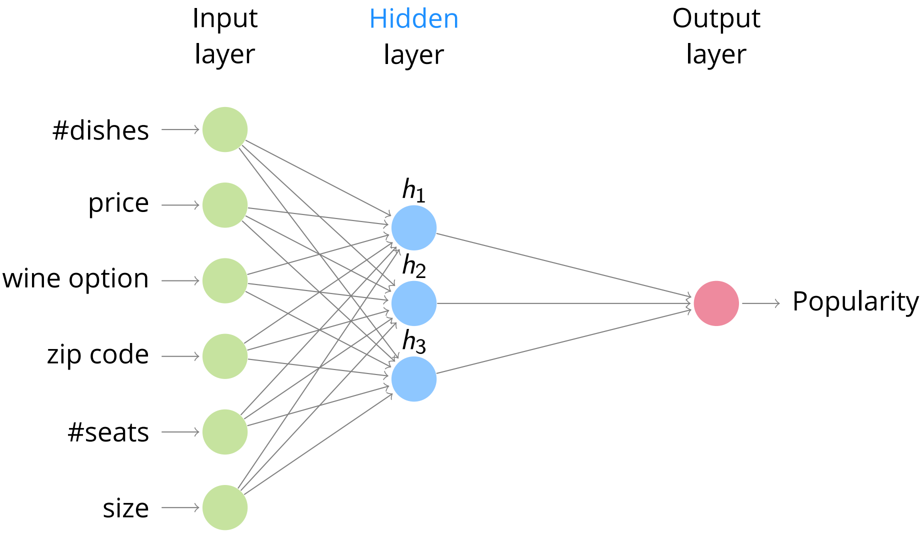
$$h_2(\text{[zip code]}) = \text{walkable}$$

$$h_3(\text{[\#seats, size]}) = \text{nosie}$$

# Predefined subproblems



# Learning intermediate features



# Neural networks

Key idea: automatically learn the intermediate features.

**Feature engineering**: Manually specify $\phi(x)$ based on domain knowledge and learn the weights:

$$f(x) = w^T \phi(x).$$

**Feature learning**: Automatically learn both the features ($K$ hidden units) and the weights:

$$h(x) = [h_1(x), \ldots, h_K(x)], \quad f(x) = w^T h(x)$$

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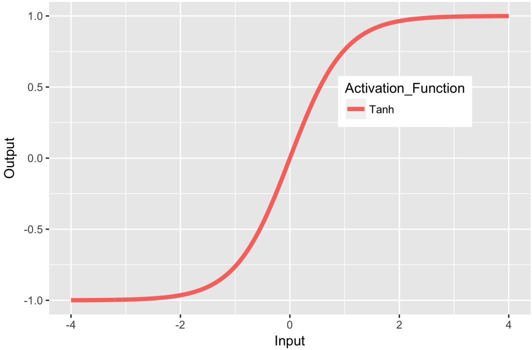
  - *Differentiable* approximations: sigmoid functions.
    - E.g., logistic function, hyperbolic tangent function, ReLU
- Two-layer neural network (one hidden layer and one output layer) with $K$ hidden units:

$$f(x) = \sum_{k=1}^{K} w_k h_k(x) = \sum_{k=1}^{K} w_k \sigma(v_k{}^T x) \tag{2}$$

# Activation Functions

- The **hyperbolic tangent** is a common activation function:

$$\sigma(x) = \tanh(x) \, .$$

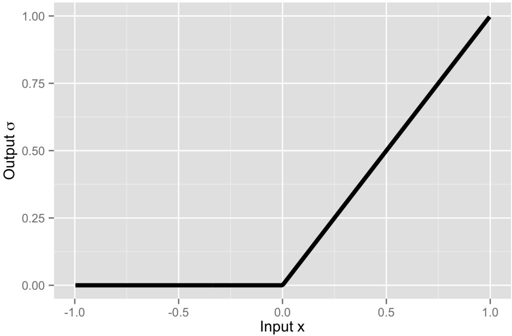


# Activation Functions

- More recently, the **rectified linear** (**ReLU**) function has been very popular:

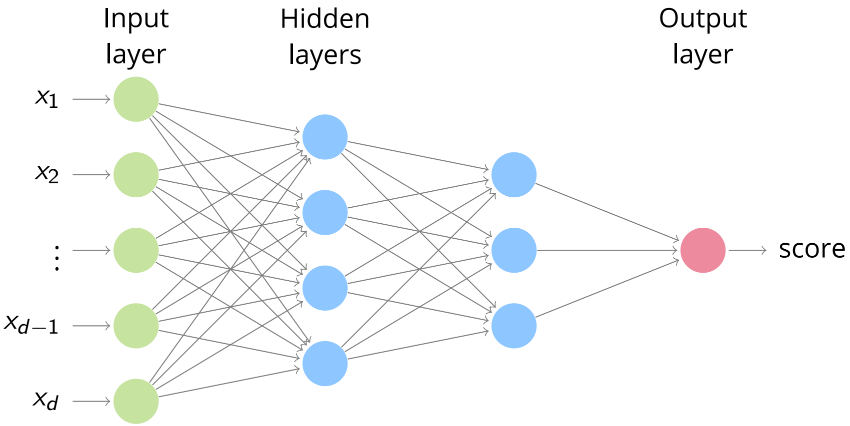
$$\sigma(x) = \max(0, x).$$

- Much faster to calculate, and to calculate its derivatives.
- Work well empirically.



# Multilayer perceptron / Feed-forward neural networks

- Wider: more hidden units.
- Deeper: more hidden layers.



# Multilayer Perceptron: Standard Recipe

- Each subsequent hidden layer takes the output $o \in \mathbb{R}^m$ of previous layer and produces

$$h^{(j)}(o^{(j-1)}) = \sigma\left(W^{(j)} o^{(j-1)} + b^{(j)}\right), \text{ for } j = 2, \ldots, L$$

  where $W^{(j)} \in \mathbb{R}^{m \times m}$, $b^{(j)} \in \mathbb{R}^m$.

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- Last layer is an *affine* mapping (no activation function):

$$a(o^{(L)}) = W^{(L+1)}o^{(L)} + b^{(L+1)},$$

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- Last layer typically gives us a score. (How to do classification?)